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Optimizing the Length of the Stores

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Optimizing the Length of the Stores
and Related Effects

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This memo is an attempt to create an accurate analytical model for the luminosity obtained in the Fermilab Tevatron during collider operation. Other people, in particular Alan Hahn and, aledgedly, Vinod Bharadwaj and Gerry Dugan, have addressed this topic and predicted an optimum store duration. The approach taken here is slightly different from Hahn's, in particular, analytic forms for the model are written down and the optima are found directly, through differentiation. Also, specifying the parameters clearly is helpful in understanding the dependencies of this phenomenology.

First, the optimum store duration is derived from first principles. Then graphs of this are made and various assumptions on the parameters are analyzed. Then some related effects are discussed. The figures are segregated at the end of this Memo.

Assumptions

This analysis does not completely parameterize the Fermilab collider. Several assumptions are made in order to (a) simplify the mathematics and (b) simplify the analysis. It does not seem to me that these assumptions, as a whole, describe a machine much different than the one we really have. Here are the most important assumptions used here:

1. The luminosity lifetime is constant throughout the store, even though the lifetime of a real store increases with time. The problem is to analytically integrate $\exp(x)$ where $x=t/(t_0+kt)$. However, as will be shown, it is quite simple to estimate the effects of the growing lifetime on all of the conclusions here.

2. The initial luminosity is determined *completely* by the stack size at the time when shot setup begins. This is still the main consideration for determining the initial luminosity of a store, although failures in other areas reduce it.

3. A specific fraction of the stack can always be extracted and then competently injected, accelerated and squeezed in the Tevatron. That is, big stacks extract to low beta with exactly the same efficiency as smaller stacks.

4. The anti-proton stacking rate falls off with stack size and this falloff is accurately parameterized, as discussed below.

5. The collider is running smoothly; a steady-state condition is assumed.

The parameters used in this analysis are summarized in the following table at the top of the next page.

Table 1, Parameters Used in the Store Length Optimization

Parameter	Description	Units	Resonable Value
T	Optimum store length	Hours	*Unknown!*
T_0	Luminosity Lifetime	Hours	10 to 30
$k_{\mathcal{L}}$	Conversion from stacksize to initial luminosity	(1E30/cm/cm/sec)/ mA	0.06 - 0.1
C	Offset for stacksize-to-init lum	1E30/cm/cm/sec	0 - 0.5
k_s	Stacking rate at t=0 (small stacksize)	mA/hour	5
S_c	Stacksize for which the stacking rate has begun to really fall off	mA	150
t_c	Time for the stack to reach the critical stacksize	hours	0.75 * S_c/k_s
t_m	Falloff for the stacking rate	hours	10
S_0	Starting stacksize	mA	-
f_s	Fraction of stack used in the shot	-	0.5
t_0	Time to make S_0	hours	-

The Analysis

The luminosity from one store can be written as follows:

$$L_{store} = \int_0^T \mathcal{L}_0 \, e^{-t/T_0} \, dt$$

where $\mathcal{L}_0$ is the initial luminosity of the store, t_0 is the luminosity lifetime and T is the length of the store. Integrating this produces:

$$L_{store} = \mathcal{L}_0 T_0 \left(1 - e^{-T/T_0}\right)$$

(Again, simplifying the integration is the reason why a constant luminosity lifetime is assumed.)

Let us assume that the initial luminosity is determined completely from the size of the anti-proton stack, S. From the data accumulated so far in run 1B, we have:

$$\mathcal{L}_0 = k_{\mathcal{L}} S + C$$

Choosing the proper values for these constants is important. If one simply performs a line fit to the data for Run 1A, then one obtains: $k_{\mathcal{L}} \approx 0.06$ (1E30 cm^{-2} sec^{-1} per milliamp) and $C \approx 0.5$ (1E30 cm^{-2} sec^{-1}). If $C \equiv 0$, then a value of 0.09 for $k_{\mathcal{L}}$ is more reasonable. It is assumed that this relationship extrapolates out for all values of S. For the stores of the week of July 25, 1994, $k_{\mathcal{L}} \geq 0.1$.

One cannot stack indefinitely. It has been observed that the stacking rate at 200 mA is about half that of a small stack [private communication from Steve Werkema]. The following formula for the stacking rate fits the observation rather well:

$$R = k_s \,/\, \cosh(t/t_c)$$

(The stacking rate is generally considered to be a function of the stack size. In order to simplify this analysis, I recast this function as one of time. *t=0* is the same as S=*0*.)

In order to fit with observations, we see that t_c is determined by:

$$t_c = 0.75\,\mathrm{S_c}/k_s$$

This quick-and-dirty estimate is necessary because the stacking rate is assumed to be a function of time.

Figure 1 shows this stacking rate for S_c = 150 mA and k_s = 5, a function which fits the data rather well. In fact, the ACNET parameters `A:SREFF` and `A:EXPSR` use exactly this parameterization.

The stacking rate can be used to determine the stack size created during the store:

$$S = S_0 + \int_{t_0}^{\mathrm{T}+t_0} \frac{k_s\,dt}{\cosh(t/t_c)}$$

$$= S_0 + 2\,k_s\,t_c\left(\arctan\left(\mathrm{e}^{(\mathrm{T}+t_o)/t_c}\right) - \arctan\left(\mathrm{e}^{t_o/t_c}\right)\right)$$

The value S_0 is the size of the stack after the shot is taken. If one uses the fraction f_s of the stack in each shot, then in the ***steady-state approximation***, we can determine the stack size at shot-setup time by:

$$S_0 = (1 - f_s)\,S$$

This initial stack size can also be determined by:

$$S_0 = \int_0^{t_0} k_s/\cosh\left(\mathrm{e}^{t/t_c}\right)dt$$

Combining the last two equations yields:

$$0 = (1 - f_s)\arctan\left(\mathrm{e}^{(\mathrm{T}+t_o)/t_c}\right) - \arctan\left(\mathrm{e}^{t_o/t_c}\right) + f_s\arctan 1$$

which is solved, numerically, for t_0. This equation expresses how long it takes to make the initial stack as a function of the length of the store. Admittedly, this is a bit obscure, but necessary in order to determine accurately the proper store length. Another way of thinking of this is as follows. Assuming f_s = 0.5 (as it generally is these days), then if 120 mA is extracted from the stack, then t_0 is the time it takes to make a 60 mA stack starting from zero. For the stack rate parameterization used here, the first 60 mA comes a lot quicker than the 60 mA from 60 to 120.

Putting this all together to get the stack size when the shot is made:

$$S = 2k_s\,t_c\left(\arctan\left(\mathrm{e}^{(\mathrm{T}+t_o)/t_c}\right) - \arctan(1)\right)$$

It is necessary to fold the solution for t_0 into this expression..

Now we are ready to write down the integrated luminosity for the week, remembering that there are (at most) 168 hours per week:

$$\mathrm{L}_{week} = \frac{168}{\mathrm{T}+t_s}\,\mathcal{L}_0\,\mathrm{T}_0\left(1 - \mathrm{e}^{-\mathrm{T}/\mathrm{T}_0}\right)$$

introducing the shot setup time, t_s. Figure 2 presents various curves for L_{week} assuming a few different sets of reasonable parameters, in particular, the luminosity lifetime, T_0. Apparently, the optimum store is about 15 to 30 hours, depending on the parameter choices. Since even a good week has a lot less than 168 hours, an appropriate uptime factor should be chosen to obtain reasonable weekly numbers.

It is desired to determine accurately the ***optimum*** length for the store, of course. So we set the differential of L_{week} with respect to T equal to zero and solve:

$$0 = \frac{-168}{(\mathrm{T}+t_s)^2}\,\mathcal{L}_0 t_0\left(1-\mathrm{e}^{-\mathrm{T}/T_0}\right) + \frac{168}{\mathrm{T}+t_s}\left(\frac{d}{d\mathrm{T}}\mathcal{L}_0\right)t_0\left(1-\mathrm{e}^{-\mathrm{T}/\mathrm{T}_0}\right) + \frac{168}{\mathrm{T}+t_s}\,\mathcal{L}_0 t_0\left(\frac{1}{t_0}\mathrm{e}^{-\mathrm{T}/\mathrm{T}_0}\right)$$

$$= -\,\mathcal{L}_0 t_0\left(1-\mathrm{e}^{-\mathrm{T}/\mathrm{T}_0}\right) + (\mathrm{T}+t_s)\,t_0\left(1-\mathrm{e}^{-\mathrm{T}/\mathrm{T}_0}\right)\left(\frac{d}{d\mathrm{T}}\,\mathcal{L}_0\right) + (\mathrm{T}+t_s)\mathcal{L}_0\left(\mathrm{e}^{-\mathrm{T}/\mathrm{T}_0}\right)$$

The differential of the initial luminosity is:

$$\frac{d}{d\mathrm{T}}\mathcal{L}_0 = k_{\mathcal{L}}\frac{d}{d\mathrm{T}}\,S$$

$$= k_{\mathcal{L}}\,\mathrm{R} = k_{\mathcal{L}}\,k_s\,/\,\cosh(t/t_c)$$

Now we are equipped to solve for T in the above equation. This has to be done numerically, but it is a rather simple algorithm. I have written this algorithm in the C++ program on my Sun computer, and the name of the program is

`/home/tomato/mccrory/collider/lifetime/optimum.cxx`

This program uses a class, `solve` (`solve.h` and `solve.cxx`) to determine the numerical solutions for T and T_0.

Data Display

Now the behavior of this function is examined for many different sets of reasonable conditions. There is certainly a lot more data presented here than can be digested quickly!

For S_c = 150 mA, k_s=5 mA/hr. This is the most realistic set of conditions.

Figure 3 shows the optimum store time as a function of the luminosity lifetime for various shot setup times. So, for a two-hour shot setup time and a 15 hour luminosity lifetime, the optimum time for a store is about 20 hours; for a 4-hour setup, the optimum store length then becomes about 22 hours.

Using the equations for the stack size as a function of T, we can plot the initial stack size for optimum length stores as a function of the luminosity lifetime for various values of t_s, Figure 4. 170-190 mA is the optimum stack size.

Figure 5: Optimum store length vs. stack rate. As the stacking rate increases, the length of the optimal store decreases since it takes less time to build up an adequate stack. As you achieve longer lifetimes, the luminosity stays high longer so that you can stack longer.

Figure 6: Optimum stack size vs. stack rate. This seems like a good curve to consult when trying to determine when to end a store. If the basic stacking rate is 5 mA/hr and the luminosity lifetime is 15 hours, then the optimal end of the store should come when the stack reaches about 180 mA.

Figure 7: Initial luminosity vs. stack rate. We should expect to get 15's to 16's (E30) for initial luminosities, when everything is running well.

Figure 8. L_{week} vs. stack rate. We could achieve 5 pb^{-1} per week. A surprising conclusion which can be drawn from this graph is that doubling the stacking rate only increases the integrated luminosity by about 50%. This is because if the stacking rate is increased without changing the capacity of the stack (the S_c parameter), then you never can reach to much bigger stack sizes before the store fades away, and, thus, larger initial luminosities. But longer stores don't hurt much.

For S_c = 300 mA. This is for the situation where the "capacity" of the Accumulator is doubled. One way of achieving this would be to build a second, identical PBar source. It is not clear if the parameterization used here would be valid in this case, but it seems like a good guess.

Figure 9: Optimum store length vs. luminosity lifetime. You can see that the bigger base stack available leads immediately to much longer store times.

Figure 10: Optimum Stack size vs T_0. Also, we can get quite larger stacks, normally around 200 mA.

Figure 11: Optimum store length vs. Stack rate. The stores are much longer--we get extra time to take advantage of the bigger Accumulator.

Figure 12: Optimum stack size vs. stack rate. The optimal stack size is relatively independent of the stacking rate, again.

Figure 13: Initial luminosity vs. stack rate. Some really big numbers.

Figure 14: L_{week} vs. stack rate. Ah! More stacking and bigger stacks really do pay off! Doubling the stacking rate (from 5 to 10 mA/hr) just about doubles the weekly integrated luminosity (from just over 6 to almost 11 pb^{-1}/week).

Reviewing Constant Luminosity Lifetime

Let's look again at the assumption that the luminosity lifetime, T_0, is constant,. Referring to Fig. 3. and using Hahn's observation of an increase in the lifetime of 0.36 hours/hour, then one could expect the luminosity at the end of an optimum store (2 hour shot setup, 15 hour lifetime, gives an optimum store duration of 20 hours) to be increased to 21.5 hours. Reading from the Fig., the optimum store length for a 21-hour lifetime is about 23 hours. So the optimum store duration for a real store would be more than 20 hours, but less than 23 hours. In general (see Fig 2), the optimum store length has a rather broad maximum, so anywhere between 18 and 24 hours should be fine. The biggest effect is probably in the integrated luminosity for the week: a lifetime of 15 hours gives about 5.4 pb^{-1}/week, whereas 20 hours gives about 6.4 pb^{-1}/week.

Typically, our initial luminosity lifetime has been around 10 hours and then it grows to be around 24 hours after a 24 hour store. This is a more serious discrepancy. Refer to TM-1901 for discussion of this issue.

Observations from This Analysis

1. The optimum store duration is determined by the rate at which the instantaneous luminosity falls off with respect to the rate at which the stack increases.

2. An increase in the luminosity lifetime helps the integrated luminosity a lot!

3. There is not much benefit from decreasing the shot setup time below the levels we normally achieve these days (2 to 4 hours).

4. Increasing the stacking rate does not help very much unless you can increase the capacity of the stack, too. (It may also help compensate for downtime.)

5. The adopted criterion of stopping the store when the stack reaches X mA is a good gauge. For us these days, using this clean analysis, X=170.

What About Downtime?

A full analysis of the effects of various sorts of downtime experienced by the collider will be presented in TM-1901.

It is possible here to estimate what is the best way to recover from a lost stack. (Unfortunately, a different set of parameters is used here, but the results should be similar: S_c=200, t_m=10, k_s=4, T_0=15 and $k_{\mathcal{L}}$=0.06 for each of the following analyses. This implies a store duration of 27 hours and a stack of 150 mA.)

There are at least two approaches to recovering from a lost stack. The first would be to stack all the way up to an optimum stack and then begin shots. The other would be to stack to some reasonable stack size (say, 80 mA), and take a shot. Then take one or two more shots on the way to reaching the optimum stack size. For the following paragraphs, the following parameters are assumed: an optimum stacksize of 150 mA and an optimum store duration of 27 hours (15 hour lifetime, 4 mA/hour stacking rate, 150 mA maximum stack). These two scenarios are summarized in the following table:

Two Scenarios for Recovering from a Lost Stack

Scenario #1 Ratchet Up to Optimum Stack

Hours	Event	Stack Size	Luminosity	Integrated L
0	Stacking reestablished	0 mA	0	0
20	80 mA achieved	80 mA	0	0
22	First shot	40 mA	5.3 E30	0
45	125 mA achieved	125 mA	1.14 E30	224 /nb
47	Second shot	62 mA	8 E30	224
80	150 mA achieved	150 mA	1.7 E30	562
				7 /nb/hour

Scenario #2: Go Directly to Optimum Stack

Hours	Event	Stack Size	Luminosity	Integrated L
0	Stacking reestablished	0 mA	0	0
40	Full stack achieved	150 mA	0	0
42	Optimum Shot	75 mA	9.5 E30	0
69	Full stack again	150 mA	1.6 E30	428 /nb
				6.2 /nb/hour

In other words, the two scenarios are close. It is the best policy (for the experiments) to use the ratcheting approach, since they don't like to wait!

Similarly, if the store is lost prematurely, it does not matter if you wait and stack up to the optimal stack size or go ahead and take a shot with what you have.

If the stack capacity were to increase well beyond the 150 mA level assumed here without any increase in the stacking rate, this sort of analysis would more strongly favor the "ratchetting" approach on a lost stack or a lost store. In other words, stack quickly to some intermediate level and then continue stacking while a store is in.

Conclusions

1. Stacking rate is not as important as the capacity of the Accumulator.
2. For the present running conditions:
   a. $18 < T < 24$ hours, optimally;
   b. We shoot from a stack of about 170 mA; initial luminosity is about 1.7 E31;
   c. We can get 4 pb^{-1}/week, maybe 5!
3. An analysis using Monte Carlo techniques to accurately simulate the effect of downtime is presented in TM-1901.

References

Alan Hahn, Fermilab Technical Memo, TM-1879, "Phenomenological Optimization of Weekly Integrated Collider Luminosity."
Elliott McCrory, Fermilab Technical Memo, TM-1901, "Modelling the Fermilab Collider to Determine Optimal Running."

Figure 3. Store Duration vs Lifetime and Setup time

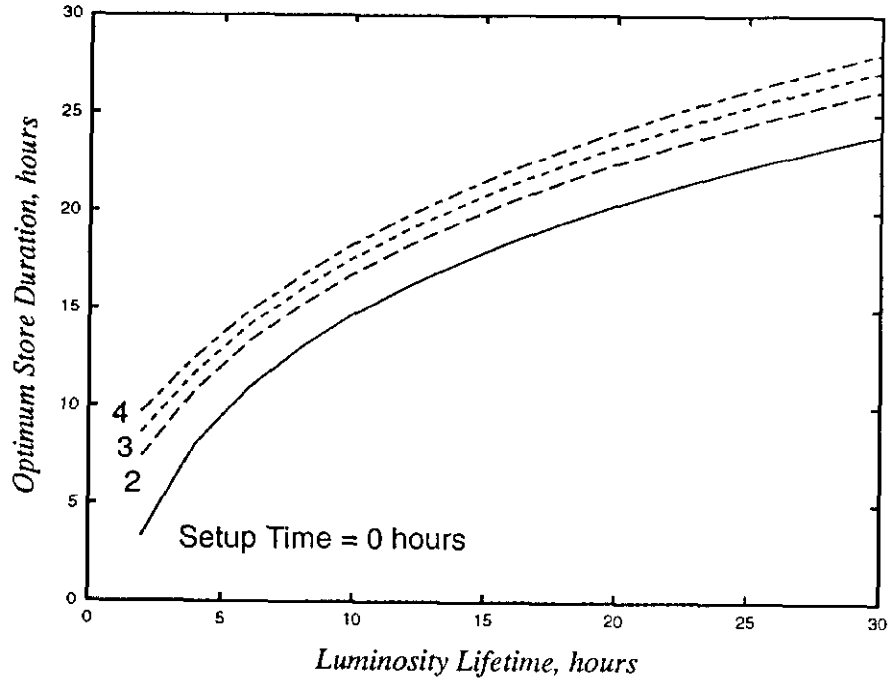

Figure 4. Optimum Stack Size vs Luminosity Lifetime and Shot Setup Time

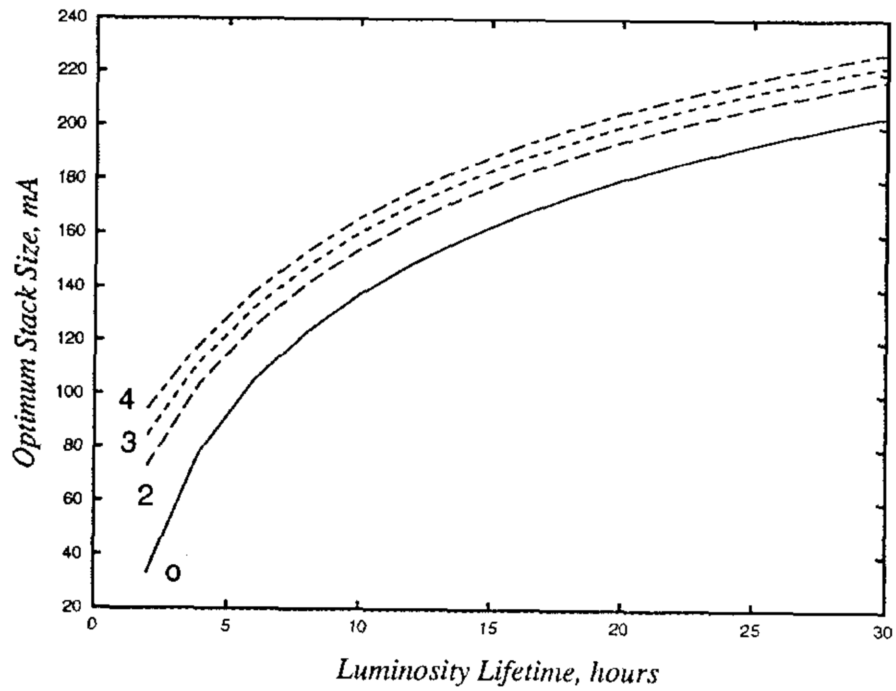

Figure 5. Store Duration vs. Stacking Rate and Luminosity Lifetime

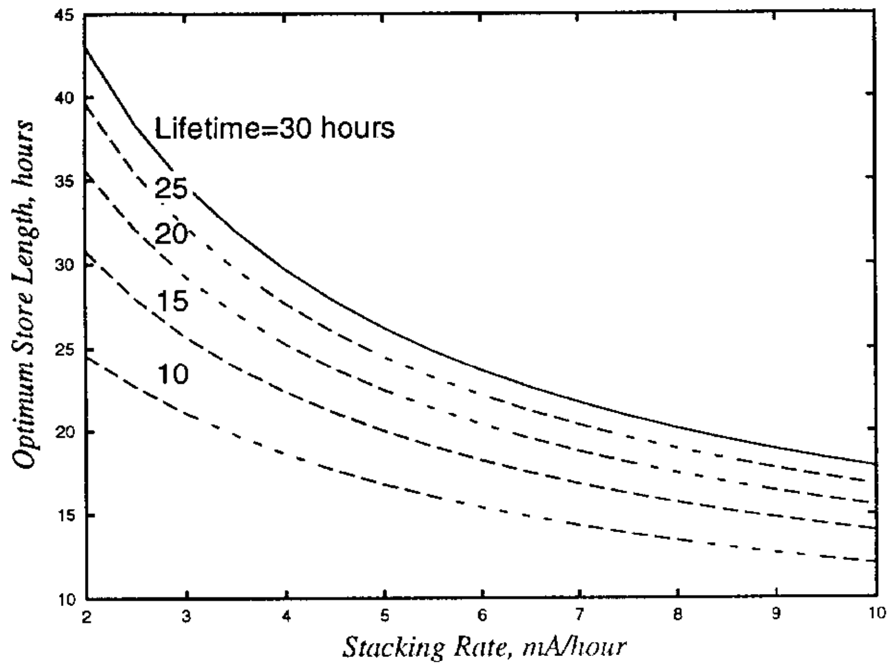

Figure 6. Optimal Stack Size vs. Stacking Rate and Luminosity Lifetime

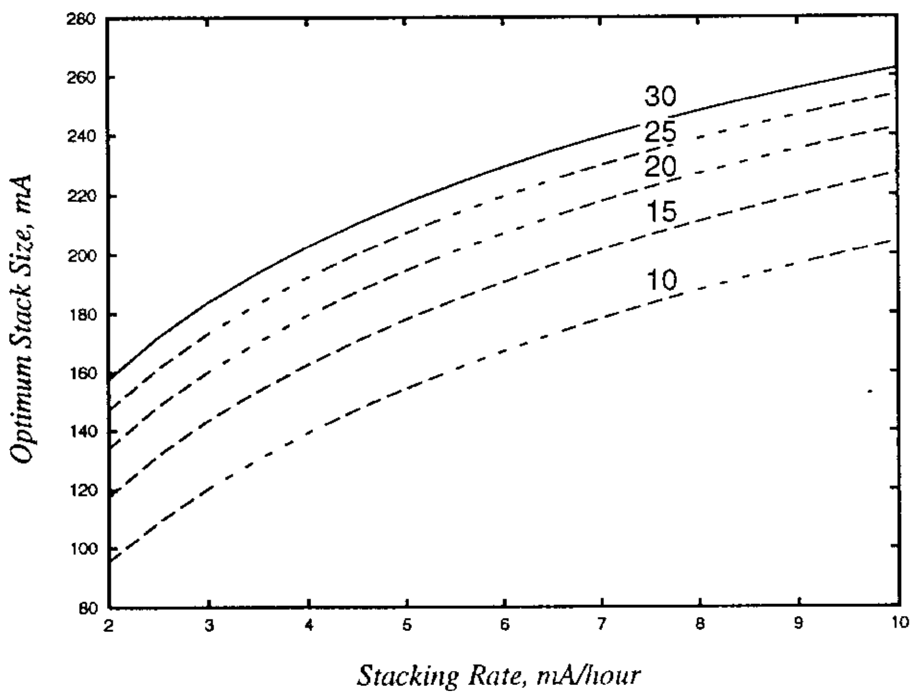

Figure 7. Initial Luminosity vs. Stacking Rate and Luminosity Lifetime

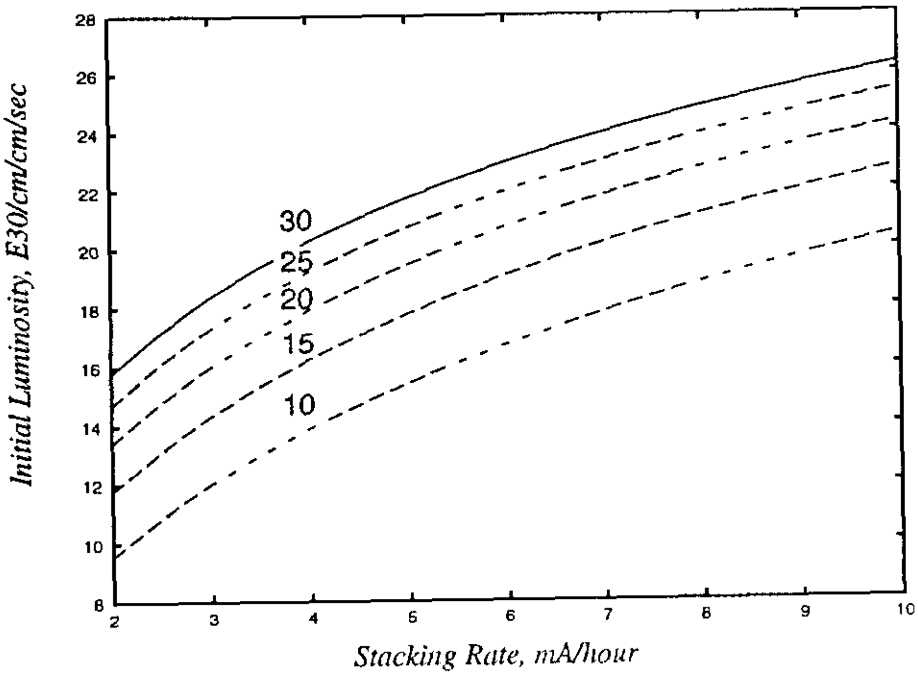

Figure 8. Weekly Integrated Luminosity vs. Stacking rate and Luminosity Lifetime

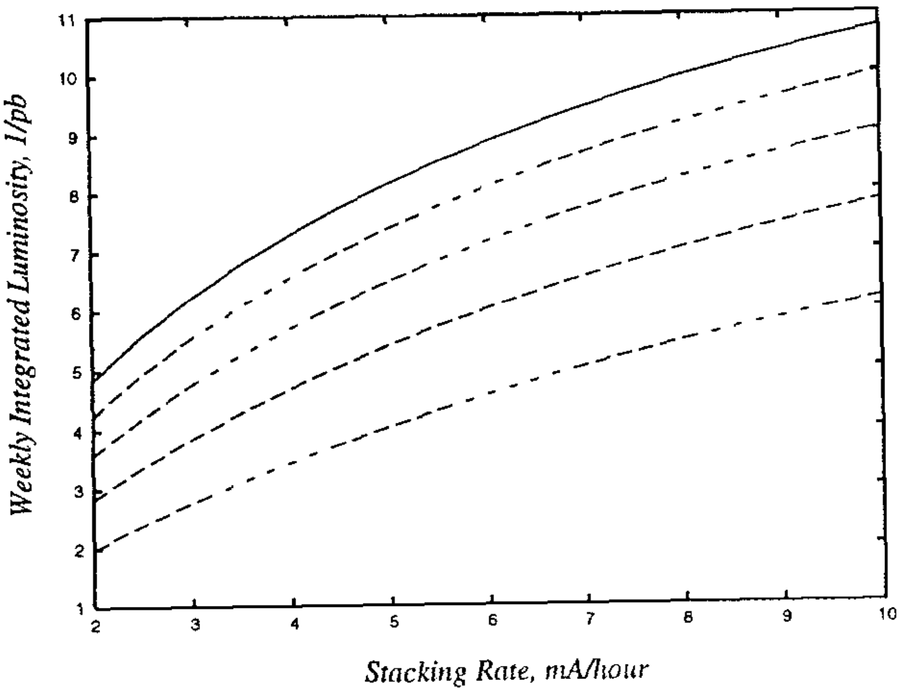

Figure 9. Sc=300 mA; Store Duration vs. Luminosity Lifetime and Setup Time

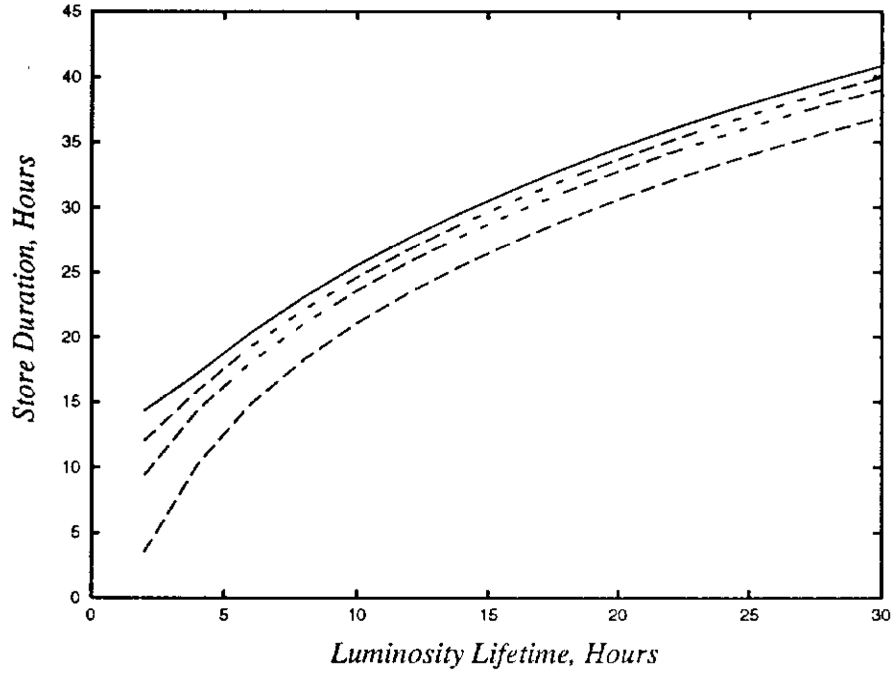

Figure 10. Sm = 300 mA; Optimum Stack Size vs. Luminosity Lifetime and Setup Time.

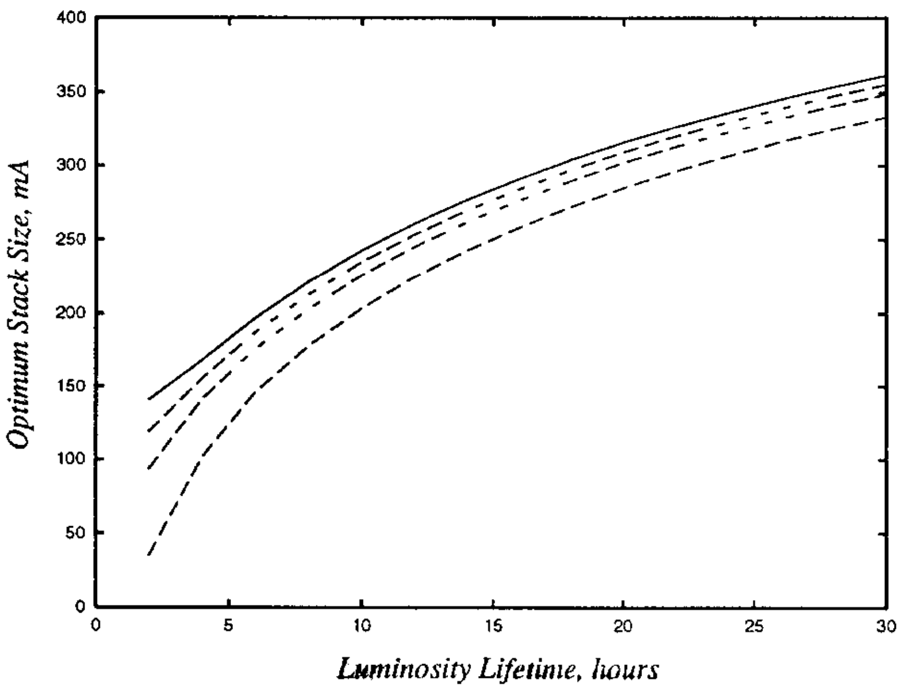

Figure 11. Sc = 300 mA; Optimum Store Duration vs. Stacking Rate and Luminosity Lifetime.

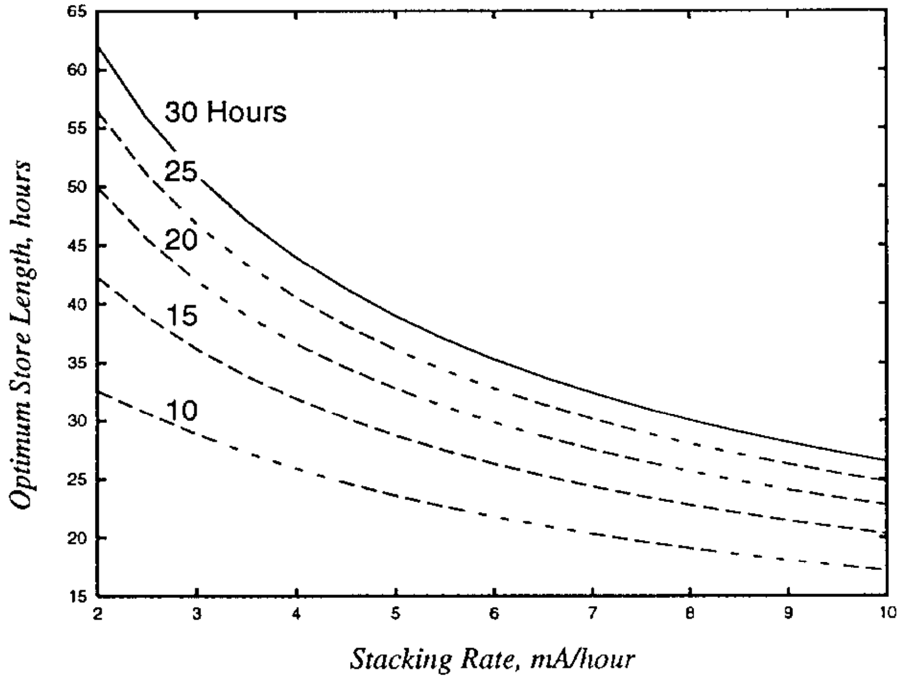

Figure 12. Sc = 300 mA; Optimum Stack Size vs. Stacking Rate and Luminosity Lifetime.

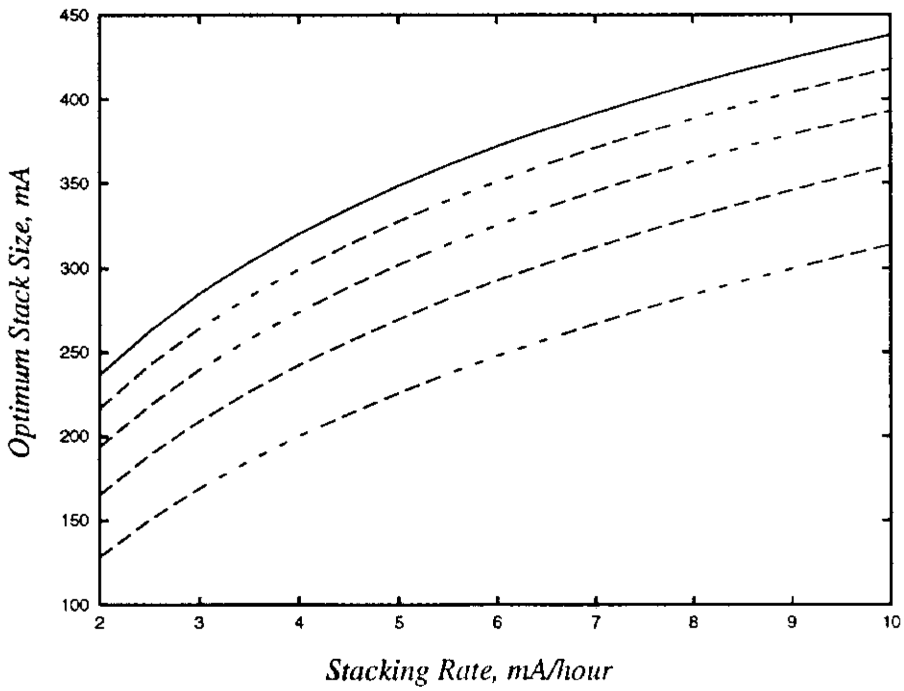

Figure 13. Sc = 300 mA; Initial Luminosity vs. Stacking Rate and Luminosity Lifetime

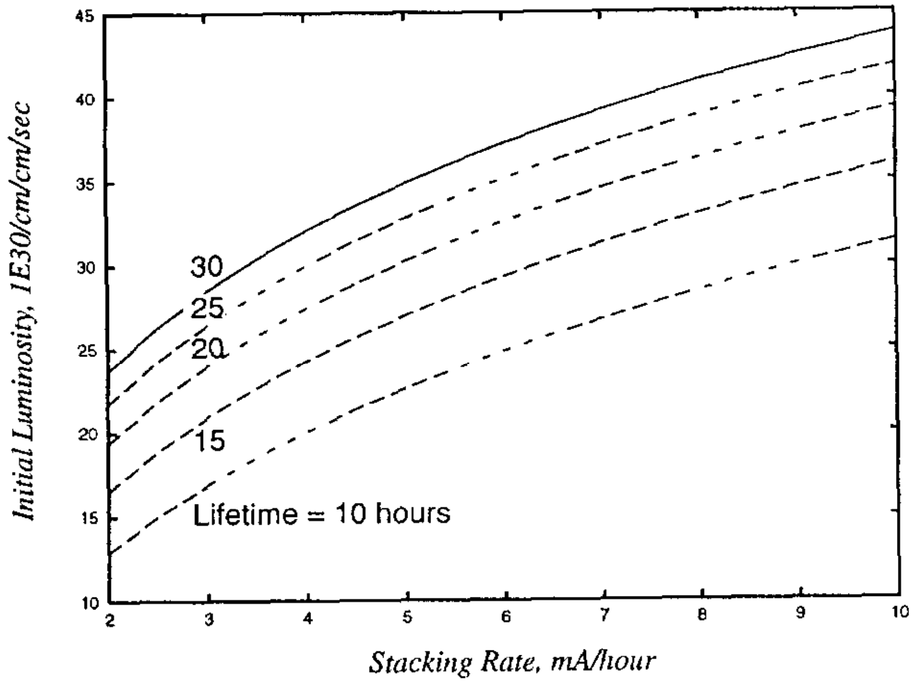

Figure 14. Sc = 300 mA; Weekly Integrated Luminosity vs. Stacking Rate and Luminosity Lifetime